Vector Fields

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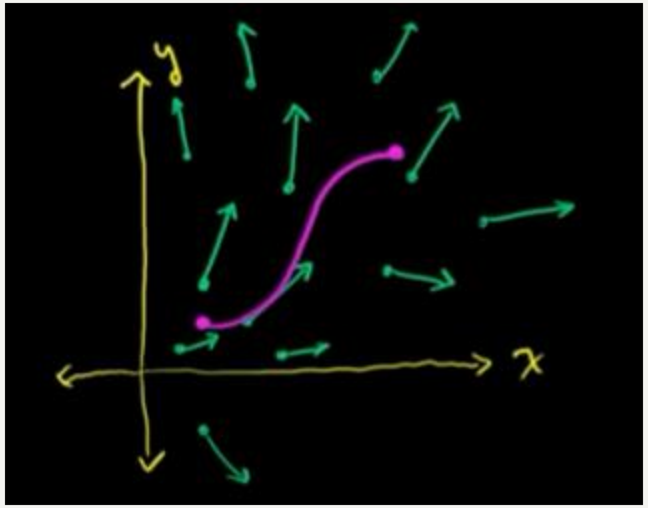
Abstract

A vector field is where a bunch of vectors are plotted to create a field of vectors allowing for the observation of flow. A vector field uses vector calculus as a base to allow calculations of these flow values. This can either be in a 2D or 3D space. Common uses of a vector field is in determining the speed and direction in which a fluid flows. This type of math is used often in Petroleum Engineering for knowing the flow of oil or injection fluids.

A Vector Field Formulation



Vector Field on a 2D plane

$\vec{f}$(x,y) = P(x,y)$\hat{\imath}$ + Q(x,y)$\hat{\jmath}$

What the path of the particle is defined by

$\vec{r}$(t) = x(t)$\hat{\imath}$ + y(t)$\hat{\jmath}$

a $\leq t \leq b$

Total work done by the Vector Field

W = $\int \vec{f} \cdot$ d $\vec{r}$

How is d $\vec{r}$ defined

d$\vec{r}$ = x′(t)dt$\hat{\imath}$ + y′(t)dt $\hat{\jmath}$

Final Formula

$$\int_{t=a}^{t=b} (P\big(x(t), y(t)\big) * x'(t)dt + Q\big(x(t), y(t)\big) * y'(t)dt)$$

Formula Application

Our formula to start with

$\vec{f}$(x,y) = y$\hat{\imath}$ - x$\hat{\jmath}$

Path formula for d $\vec{r}$

$\vec{r}$(t) = cos(t) $\hat{\imath}$ + sin(t) $\hat{\jmath}$

The d(derivative) of $\vec{r}$

$\vec{r}$(t) = -sin(t) dt $\hat{\imath}$ + cos(t) dt $\hat{\jmath}$

Now multiply $\vec{f} \cdot$ d $\vec{r}$

(sin(t) $\hat{\imath}$ - cos(t) $\hat{\jmath}$)(-sin(t) dt $\hat{\imath}$ + cos(t) dt $\hat{\jmath}$)

Final Equation

$-\int_0^{2\pi}(sin^2 t + cos^2 t)dt = -\int_0^{2\pi}(1dt)$ = -[t]$_0^{2\pi}$ = -(2$\pi - 0) = -2\pi$

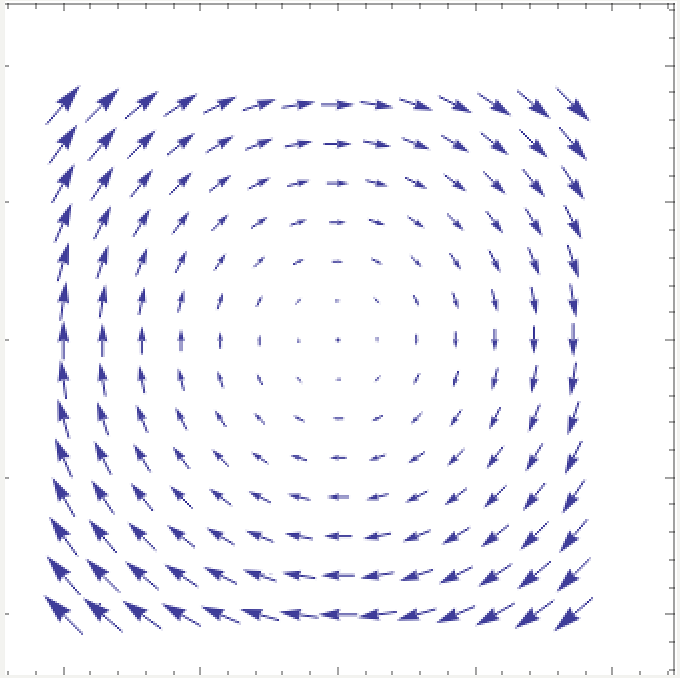
Our constants

x(t) = cos t

y(t) = sin t

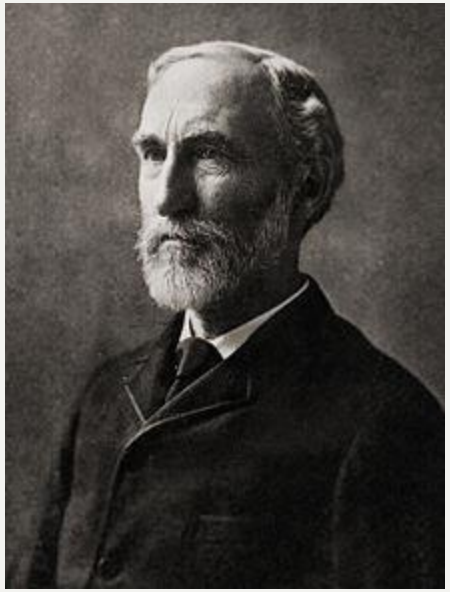
$0 \leq t \leq 2\pi$

W = $\int \vec{f} \cdot$ d $\vec{r}$

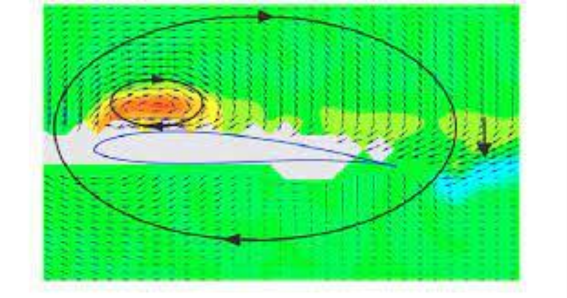


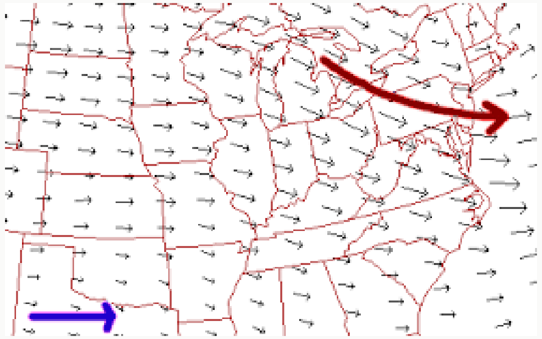
Who invented Vector Fields?

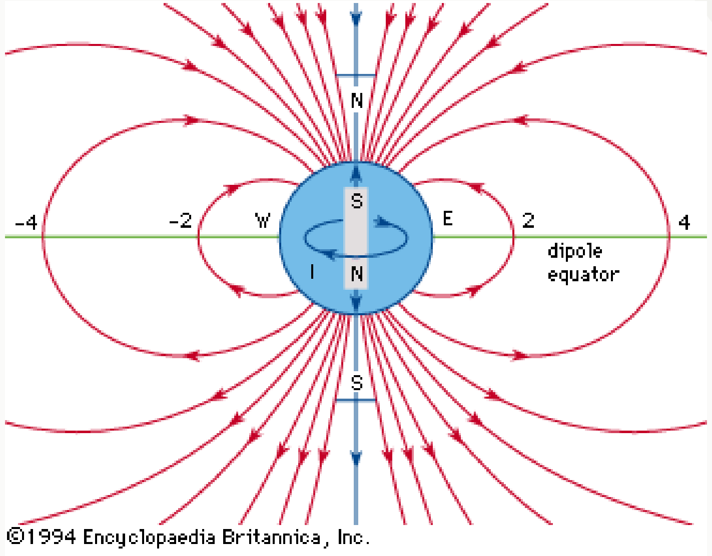
Vector calculus and its sub objective Vector Fields was invented by two men J. Willard Gibbs and Oliver Heaviside at the end of the 19th century. This allowed scientists and mathematicians to calculate such things as speed and direction from a graph. It also allows for a great visualization of what is happening with these numbers.



J. Willard Gibbs and Oliver Heaviside







Graphical Examples of Vector Fields

Conclusion

Prior to researching about Vector Fields I had no idea how math, in particular calculus, played a role in figuring this stuff out. I have always seen some variant of a vector field before but never thought to much of it. It is amazing how we can figure out how a particle will react by using vector formulas to then display them as readable graphs. Calculus truly allows us as a species to continue developing and progressing into the future.

Work Cited

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